

Correlator of Topological Charge Densities in Instanton Model in QCD.

B.L. Ioffe* and A.V. Samsonov†

Institute of Theoretical and Experimental Physics
B.Chermushkinskaya 25, 117218, Moscow, Russia

Abstract

The QCD sum rule for the correlator of topological charge densities $\chi(Q^2)$ and related to it longitudinal part of the correlator of singlet axial currents is considered in the framework of instanton model. The coupling constant $f_{\eta'}$ of η' -meson with the singlet axial current is determined. Its value appears to be in a good coincidence with the value determined recently from the connection of the part of proton spin Σ , carried by u, d, s quarks, with the derivative of QCD topological susceptibility $\chi'(0)$. From the same sum rule $\eta - \eta'$ mixing angle θ_8 is found in the framework of two mixing angles model. The value of θ_8 is close to that found in the chiral effective theory. The correlator of topological charge densities $\chi(Q^2)$ at large Q^2 is calculated and it was found that its Q^2 -dependence matches well with Q^2 -dependence at low Q^2 determined by the known $\chi'(0)$ and by contributions of π - and η -mesons.

*e-mail:ioffe@vitep5.itep.ru

†e-mail:sams@heron.itep.ru

1 Introduction.

Recently the vacuum expectation value of singlet axial current f_0^2 induced by external singlet axial field A_μ has been found [1]:

$$\langle 0 | j_{\mu 5} | 0 \rangle_A = 3f_0^2 A_\mu. \quad (1)$$

In (1) $j_{\mu 5}$ is the singlet quark current

$$j_{\mu 5}(x) = \sum_q \bar{q}(x) \gamma_\mu \gamma_5 q(x), \quad q = u, d, s. \quad (2)$$

The term

$$\Delta L = j_{\mu 5} A_\mu \quad (3)$$

was added to QCD Lagrangian, where A_μ is a constant in space and time singlet axial field and the limit of weak A_μ field was considered. It was found in the limit of massless u, d, s quarks [1, 2]:

$$f_0^2 = (2.8 \pm 0.7) \cdot 10^{-2} \text{ GeV}^2. \quad (4)$$

This result was obtained, constructing the QCD sum rule in the external field A_μ to determine the part of proton spin Σ carried by u, d, s quarks and related to the proton matrix element of the current $j_{\mu 5}$:

$$2ms_\mu \Sigma = \langle p, s | j_{\mu 5} | p, s \rangle, \quad (5)$$

where s_μ is the proton spin 4-vector, m is the proton mass. The sum rule for Σ essentially depends on f_0^2 and the numerical result (4) comes in two ways, resulting in the same value:

1) from the requirement of consistency of phenomenological and calculated in QCD sides of the sum rule as functions of the Borel parameter M^2 ;

2) by using for Σ its experimental value, $\Sigma = 0.3 \pm 0.1$.

As was shown in [1], in the limit of massless u, d, s quarks f_0^2 is related to the first derivative $\chi'(0)$ of the correlator of topological charge densities $Q_5(x)$

$$\chi(q^2) = i \int d^4x e^{iqx} \langle 0 | T \{ Q_5(x), Q_5(0) \} | 0 \rangle, \quad (6)$$

$$Q_5(x) = \frac{\alpha_s}{8\pi} G_{\mu\nu}^n(x) \tilde{G}_{\mu\nu}^n(x), \quad (7)$$

where $G_{\mu\nu}^n(x)$ is gluonic field strength, $\tilde{G}_{\mu\nu}^n(x)$ is its dual, $\tilde{G}_{\mu\nu}^n = (1/2)\varepsilon_{\mu\nu\lambda\sigma} G_{\lambda\sigma}^n$:

$$f_0^2 = 12\chi'(0). \quad (8)$$

As follows from (4),

$$\chi'(0) = (2.3 \pm 0.6) \cdot 10^{-3} \text{ GeV}^2. \quad (9)$$

Let us remind the derivation of (8). Using (3) we can write:

$$\langle 0 | j_{\mu 5} | 0 \rangle_A = \lim_{q \rightarrow 0} i \int d^4x e^{iqx} \langle 0 | T \{ j_{\nu 5}(x), j_{\mu 5}(0) \} | 0 \rangle A_\nu \equiv \lim_{q \rightarrow 0} P_{\mu\nu}(q) A_\nu. \quad (10)$$

The general structure of $P_{\mu\nu}(q)$ is

$$P_{\mu\nu}(q) = -P_L(q^2)\delta_{\mu\nu} + P_T(q^2)(-\delta_{\mu\nu}q^2 + q_\mu q_\nu). \quad (11)$$

Because of anomaly there are no massless states in the spectrum of singlet polarization operator $P_{\mu\nu}$ even for massless quarks. $P_{T,L}(q^2)$ also have no kinematical singularities at $q^2 = 0$. Therefore, the nonvanishing value $P_{\mu\nu}(0)$ comes entirely from $P_L(q^2)$. Multiplying $P_{\mu\nu}(q)$ by $q_\mu q_\nu$ and using the anomaly condition

$$\partial_\mu j_{\mu 5}(x) = 2N_f Q_5(x) + 2i \sum_q m_q \bar{q}(x) \gamma_5 q(x) \quad (12)$$

(N_f is the number of flavours, $N_f = 3$), in the limit of massless quarks we get:

$$q_\mu q_\nu P_{\mu\nu}(q) = -P_L(q^2)q^2 = 36\chi(q^2). \quad (13)$$

As is known [3], $\chi(0) = 0$ if there is at least one massless quark. (8) follows directly from (1), (10), (11), (13). According to (1) we have also:

$$f_0^2 = -(1/3)P_L(0). \quad (14)$$

The attempt to determine f_0^2 directly by constructing a special QCD sum rule for this aim was performed in [4]. However, this attempt failed: it was found that the operator product expansion (OPE) used in the sum rule does not converge on the accounted in calculation terms – the unaccounted higher order terms of OPE should be of importance. In the present paper we use the instanton model [5] (as review see [6]) for calculation of these higher order terms. The idea that instantons give the main contribution to the longitudinal part $P_L(q^2)$ of the correlator of singlet axial current $P_{\mu\nu}(q)$ at intermediate $|q^2| \sim 1 \text{ GeV}^2$ is not new – it was suggested as early as in 1979 [7, 8]. In [7] it was argued that the appearance of η - and η' -mesons as almost pure octet and singlet states in $SU(3)$ flavour symmetry, i.e. the large mixing of $\bar{u}u + \bar{d}d$ and $\bar{s}s$ in this channel, cannot be described by perturbative QCD and may be attributed only to dominating instanton contribution.

The sum rule for direct determination of f_0^2 is constructed. At standard parameters of the instanton model the value of f_0^2 found from the sum rule is in a good agreement with (4).

For massless quarks the phenomenological side of the sum rule is saturated by η' -meson contribution (plus contributions of excited states, approximated by continuum). The strange quark mass m_s may be also accounted in the sum rule. In this case the contribution of η -meson also comes into play and $\eta - \eta'$ mixing angle can be found from the analysis. In the model of two mixing angles [9] the value of the largest angle θ_8 is determined and appears to be in a good agreement with the values found from the chiral theory and phenomenology [9, 10, 11].

In the framework of the same instanton model the q^2 -dependence of the topological charge densities correlator $\chi(Q^2)$ is determined at space-like $Q^2 = -q^2 > 0$. At intermediate $Q^2 \sim 1\text{GeV}^2$ it matches well with the curve of the $\chi(Q^2)$ behaviour at low Q^2 , found in [12] on the basis of (9) and contributions of Goldstone bosons π^0 and η .

2 The sum rule.

Strictly speaking, the quantity f_0^2 given by (1) is defined as nonperturbative part of the induced by the external field A_μ vacuum expectation value $\langle 0 | j_{\mu 5} | 0 \rangle_A$ with perturbative contribution subtracted. The numerical value (4) corresponds just to such definition. The reason for this definition is that in the sum rule for Σ , from which the value of f_0^2 (4) was determined, all perturbative contributions were accounted explicitly and only non-perturbative part of $\langle 0 | j_{\mu 5} | 0 \rangle_A$ was parameterized by unknown constant f_0^2 . Similarly, $\chi(q^2)$ in (6) and (8) has the meaning of nonperturbative part of the correlator of topological charge densities. Such definition is physically reasonable, since the perturbative part of (6) is badly divergent, strongly depends on renormalization scheme, and therefore has no physical meaning. The same statement refers to perturbative contribution to f_0^2 . Such separation of perturbative and nonperturbative contributions allows one to avoid any uncertainties in the sum rule for the physically measurable quantity Σ .

The idea of determination of f_0^2 or, what is equivalent, proportional to f_0^2 quantity $P_L(0)$ was suggested in [4]. In short, it was the following.

The imaginary part of $P_L(q^2)$ is represented by contributions of the lowest resonance – η' -meson – and continuum:

$$\text{Im}P_L(q^2) = 3\pi\tilde{f}_{\eta'}^2 m_{\eta'}^2 \delta(q^2 - m_{\eta'}^2) + \beta(q^2)\theta(q^2 - s_0). \quad (15)$$

Here $\tilde{f}_{\eta'}$ is the coupling of η' -meson with the singlet axial current

$$\langle 0 | j_{\mu 5} | \eta' \rangle = i\sqrt{3}\tilde{f}_{\eta'} q_\mu \quad (16)$$

in the limit of massless u, d, s quarks (q_μ is η' momentum). The second term in the right hand side (rhs) of (15) represents the contribution of continuum, s_0 is the continuum threshold. The continuum contribution corresponds to gluonic bare loop in the correlator (6) and is equal to

$$\beta(q^2) = \frac{9\alpha_s^2}{8\pi^3} q^2. \quad (17)$$

In order to get the nonperturbative part of $\text{Im}P_L$ the perturbative part equal to $\beta(q^2)$ must be subtracted from (15) what gives:

$$\text{Im}P_L(q^2)_{\text{nonp}} = 3\pi\tilde{f}_{\eta'}^2 m_{\eta'}^2 \delta(q^2 - m_{\eta'}^2) - \beta(q^2)\theta(s_0 - q^2). \quad (18)$$

As was shown in [4], the nonperturbative part of $P_L(0)$ is given by

$$P_L(0)_{\text{nonp}} = -3\tilde{f}_{\eta'}^2 + \frac{1}{\pi} \int_0^{s_0} \frac{\beta(s)}{s} ds. \quad (19)$$

Therefore the problem reduces to determination of the coupling constant \tilde{f}_η^2 . This can be done by standard technique of the QCD sum rule approach. Let us write OPE for $P_L(Q^2) = (36/Q^2)\chi(Q^2)$ at large Q^2 . We account the instanton contribution in the instanton liquid approximation. (The instanton contribution was not accounted in [2].) The OPE for $P_L(Q^2)$ has the form [8]:

$$\begin{aligned}
-P_L(Q^2) = & \frac{9\alpha_s^2}{8\pi^4} Q^2 \ln \frac{Q^2}{\mu^2} + \frac{9\alpha_s}{4\pi} \frac{1}{Q^2} \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle + \\
& + \frac{9\alpha_s^2}{2\pi^2} \frac{1}{Q^4} g f^{abc} \langle 0 | G_{\mu\alpha}^a G_{\alpha\beta}^b G_{\beta\mu}^c | 0 \rangle + \\
& + \frac{9\alpha_s^3}{2\pi Q^6} f^{abc} f^{ade} \langle 0 | G_{\mu\nu}^b G_{\alpha\beta}^c G_{\mu\nu}^d G_{\alpha\beta}^e + 10 G_{\mu\alpha}^b G_{\alpha\nu}^c G_{\mu\beta}^d G_{\beta\nu}^e | 0 \rangle + \\
& + 18 Q^2 \int d\rho n(\rho) \rho^4 K_2^2(Q\rho) .
\end{aligned} \tag{20}$$

In (20) operators up to dimension 8 were accounted. The first term in the rhs of (20) is the bare loop contribution, the last one is the contribution of instanton [7, 8], $K_2(x)$ is the McDonald function, ρ is the instanton size and $n(\rho)$ is the instanton density. (In our normalization anti-instantons are accounted in the coefficient 18 in front of the last term in (20) and $n(\rho)$ represents the instanton density.) For $n(\rho)$ we use the Shuryak model [5, 6] of instanton density:

$$n(\rho) = n_0 \delta(\rho - \rho_c) . \tag{21}$$

As was demonstrated by Shuryak and his collaborators [6], this model well describes many hadronic correlators in QCD. For numerical values of parameters in (21) we choose: $n_0 = 0.75 \cdot 10^{-3} \text{ GeV}^4$, $\rho_c = 1.5 \text{ GeV}^{-1}$, inside allowed by this model limits. Particularly, at this n_0 the standard value of gluonic condensate

$$\langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle = 0.012 \text{ GeV}^4 \tag{22}$$

may be attributed entirely to instantons. In order to estimate 8-dimensional gluonic condensate we assume the factorization hypothesis – the saturation by vacuum intermediate states. Then [8]:

$$\begin{aligned}
& f^{abc} f^{ade} \langle 0 | G_{\mu\nu}^b G_{\alpha\beta}^c G_{\mu\nu}^d G_{\alpha\beta}^e + 10 G_{\mu\alpha}^b G_{\alpha\nu}^c G_{\mu\beta}^d G_{\beta\nu}^e | 0 \rangle = \\
& = \frac{15}{16} \langle 0 | G_{\mu\nu}^n G_{\mu\nu}^n | 0 \rangle^2 .
\end{aligned} \tag{23}$$

It should be mentioned that the calculation of the same term in the instanton model would give quite different result:

$$(2^{11} \cdot 3\pi/7) n_0 \rho_c^{-4} , \tag{24}$$

which is by an order of magnitude larger than (23) at accepted model parameters. This fact is not surprising. Indeed, for the gluonic condensate with k gluonic fields on dimensional ground we would have:

$$\langle 0 | G^k | 0 \rangle \sim \int n(\rho) \frac{1}{\rho^{2k-4}} d\rho, \quad (25)$$

and the integral (25) diverges at high enough k at any physical $n(\rho)$. Therefore, one may expect that the instanton model overestimates the value of 8-dimensional gluonic condensate and accept the estimation (23) based on factorization hypothesis. This estimation is supported by the analysis of the sum rules for heavy quarkonia with the account of 8-dimensional operators [13] where the factorization hypothesis was used. Much larger values of 8-dimensional gluonic condensates would contradict the analysis in [13]. Of course, two or, may be, even 3 times larger values as (23) are not excluded, but, luckily, the contribution of these condensates to the sum rule is small and even its 3 times increasing does not influence seriously the result. For the 6-dimensional gluonic condensate there is no other independent estimation as given by instanton model [8]:

$$\frac{g^3}{12\pi^2} f^{abc} \langle 0 | G_{\mu\nu}^a G_{\nu\alpha}^b G_{\alpha\mu}^c | 0 \rangle = \frac{4}{5} \frac{1}{\rho_c^2} \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle. \quad (26)$$

The phenomenological representation of $P_L(Q^2)$ follows from (15). Equating these two representation and applying the Borel transformation to both sides of this equality we get the sum rule:

$$3\tilde{f}_{\eta'}^2 m_{\eta'}^2 e^{-m_{\eta'}^2/M^2} = \frac{9\alpha_s^2}{8\pi^4} M^4 E_1\left(\frac{s_0}{M^2}\right) + \frac{9\alpha_s}{4\pi} \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle \left(1 + \frac{\varepsilon}{M^2}\right) + \frac{135}{64} \frac{\pi\alpha_s}{M^4} \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle^2 + 18n_0\rho_c^4 \mathcal{B}_{M^2} Q^2 K_2^2(Q\rho_c), \quad (27)$$

$$E_1(x) = 1 - (1+x)e^{-x}, \quad (28)$$

where \mathcal{B}_{M^2} means Borel transform. In (27) ε corresponds to contribution of 6-dimensional gluonic condensate. The instanton model estimation (26) gives $\varepsilon = 2.2 \text{ GeV}^2$. Since we expect that the instanton model overestimates 6-dimensional gluonic condensate also, we put $\varepsilon = 1 \text{ GeV}^2$ and include possible uncertainty in the error. The Borel transformation of McDonald function can be done by using its asymptotic expansion, what leads to (see [14]):

$$18n_0\rho_c^4 \mathcal{B}_{M^2} Q^2 K_2^2(Q\rho_c) \approx 9n_0 M^3 \rho_c^3 \sqrt{\pi} e^{-M^2\rho_c^2} \left(M^2 \rho_c^2 + \frac{13}{4} + \frac{165}{32} \frac{1}{M^2 \rho_c^2} \right). \quad (29)$$

In our M^2 domain the next terms of the expansion are small. In order to verify this fact numerical calculation was done, using integral representation of McDonald function.

The results of the calculation of $\tilde{f}_{\eta'}^2$ according to the sum rule (27) are plotted in Fig.1. (It was put $\Lambda_{QCD} = 200 \text{ MeV}$, α_s was calculated in the leading order approximation.) The standard estimation procedure of the M^2 interval where the sum rule is reliable – the requirement that highest order terms of OPE are small – does not work here, because the instanton contribution dominates, it comprises about 75–80% of the total. So we go to physical arguments. We put the continuum threshold $s_0 = 2.5 \text{ GeV}^2$ close to the position of the second resonance with η' quantum numbers, $\eta'(1440)$ (probably, $\eta(1295)$ the belongs

to octet), and require that the continuum contribution to bare loop does not exceed $\sim 50\%$. As a low limit of M^2 interval we choose the M^2 value where the M^2 -dependence starts to rise steeply. These requirements result in $1.2 \lesssim M^2 \lesssim 1.6 \text{ GeV}^2$. In this interval M^2 -dependence is not very strong and we estimate $\tilde{f}_{\eta'}^2 \approx (2.4 \pm 0.6) \cdot 10^{-2} \text{ GeV}^2$. (The error includes 15% possible variation of ρ_c .) The contribution of the second term in the *rhs* of (19) is negligibly small. From (14) and (8) we have finally:

$$f_0^2 = (2.4 \pm 0.6) \cdot 10^{-2} \text{ GeV}^2, \quad (30)$$

$$\chi'(0) = (2.0 \pm 0.5) \cdot 10^{-3} \text{ GeV}^2, \quad (31)$$

in a good agreement with (4) and $\chi'(0)$ value found in [1], $\chi'(0) = (2.3 \pm 0.6) \cdot 10^{-3} \text{ GeV}^2$.

3 The account of strange quark mass. $\eta' - \eta$ mixing angle.

Consider the polarization operator $P_L(q^2)$ with the account of the strange quark mass m_s and determine the coupling constant $f_{\eta'}$ of physical η' . The u - and d -quark masses are disregarded as before. Using the definition of $P_{\mu\nu}(q)$ and (12) we have:

$$\begin{aligned} -P_L(q^2)q^2 &= q_\mu q_\nu P_{\mu\nu}(q) = i \int d^4x e^{iqx} \langle 0 | T \{ 2N_f Q_5(x), 2N_f Q_5(0) \} + \\ &+ T \{ 2N_f Q_5(x), 2im_s \bar{s}(0) \gamma_5 s(0) \} + T \{ 2im_s \bar{s}(x) \gamma_5 s(x), 2N_f Q_5(0) \} - \\ &- 4m_s^2 T \{ \bar{s}(x) \gamma_5 s(x), \bar{s}(0) \gamma_5 s(0) \} | 0 \rangle + 4m_s \langle 0 | \bar{s}(0) s(0) | 0 \rangle. \end{aligned} \quad (32)$$

The last term in (32) is caused by the equal-time commutator. Perform the OPE in the *rhs* of (32). Then in comparison with (20) three additional terms appear: the equal-time commutator term, proportional to m_s^2 term, corresponding to bare loop of strange quarks, and arising from the second and third terms in (32) the term, proportional to quark-gluon condensate [15]:

$$-g \langle 0 | \bar{s} \sigma_{\mu\nu} (\lambda^n/2) G_{\mu\nu}^n s | 0 \rangle = m_0^2 \langle 0 | \bar{s} s | 0 \rangle, \quad (33)$$

where $m_0^2 = 0.8 \text{ GeV}^2$ was determined in [16]. After Borel transformation the *rhs* of the sum rule is now:

$$R(M^2) - 4m_s \langle 0 | \bar{s} s | 0 \rangle + \frac{3m_s^2}{2\pi^2} M^2 E_0 \left(\frac{s_0}{M^2} \right) L^{-8/9} - 6\alpha_s \frac{m_s m_0^2}{\pi M^2} \langle 0 | \bar{s} s | 0 \rangle, \quad (34)$$

where $R(M^2)$ is the *rhs* of (27) and

$$E_0(x) = 1 - e^{-x}, \quad (35)$$

$$L = \ln(M^2/\Lambda^2)/\ln(\mu^2/\Lambda^2). \quad (36)$$

The factor $L^{-8/9}$ accounts quark mass anomalous dimension.

It is useful to consider also the correlator $P_{\mu\nu}(q)$ in the case when one of the currents is still $j_{\mu 5}(x)$, but the other is the current of u - and d -quarks: $\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d$. In this case the rhs of the sum rule is equal to

$$\frac{2}{3}R(M^2) - 2\alpha_s \frac{m_s m_0^2}{\pi M^2} \langle 0 | \bar{s}s | 0 \rangle. \quad (37)$$

In the phenomenological – left hand side (lhs) of the sum rule – η' - and η -mesons are contributing and their mixing must be accounted. We adopt the two mixing angles model [9], which is based on the low energy chiral effective theory and describes the experimental data better [9, 10] than the one mixing angle model. In this model the couplings of η - and η' -mesons f_η and $f_{\eta'}$ to octet and singlet axial currents are related to the couplings of fictitious octet and singlet pseudoscalar states f_8 and f_1 by:

$$\begin{aligned} f_\eta^8 &= f_8 \cos\theta_8 & f_\eta^1 &= -f_1 \sin\theta_1 \\ f_{\eta'}^8 &= f_8 \sin\theta_8 & f_{\eta'}^1 &= f_1 \cos\theta_1 \end{aligned} \quad (38)$$

The η' - and η -meson contributions to $P_L(q^2)$ can be easily calculated in this model. It is convenient to present them separately for the cases when one of the currents is $\bar{s}\gamma_\mu\gamma_5 s$ or $\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d$ (the other is always $j_{\mu 5}$). Instead of the lhs of (27) we have now:

for $\bar{s}\gamma_\mu\gamma_5 s$ current:

$$\begin{aligned} & m_{\eta'}^2 e^{-m_{\eta'}^2/M^2} f_1^2 [\cos^2\theta_1 - \sqrt{2}(f_8/f_1) \sin\theta_8 \cos\theta_1] + \\ & + m_\eta^2 e^{-m_\eta^2/M^2} f_1^2 [\sin^2\theta_1 + \sqrt{2}(f_8/f_1) \sin\theta_1 \cos\theta_8]; \end{aligned} \quad (39)$$

for $\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d$ current:

$$\begin{aligned} & m_{\eta'}^2 e^{-m_{\eta'}^2/M^2} f_1^2 [2\cos^2\theta_1 + \sqrt{2}(f_8/f_1) \sin\theta_8 \cos\theta_1] + \\ & + m_\eta^2 e^{-m_\eta^2/M^2} f_1^2 [2\sin^2\theta_1 - \sqrt{2}(f_8/f_1) \cos\theta_8 \sin\theta_1]. \end{aligned} \quad (40)$$

Taking the sum of (39), (40), putting $\theta_1 = \theta_8 = 0$ and equating it to (34) at $m_s = 0$, we get the previous result with $f_1 = f_0$. At nonzero m_s the mixing angles must be accounted and for the sum of (39), (40) we get:

$$\begin{aligned} & 3m_{\eta'}^2 f_1^2 e^{-m_{\eta'}^2/M^2} \left[\cos^2\theta_1 + \frac{m_\eta^2}{m_{\eta'}^2} e^{(m_{\eta'}^2 - m_\eta^2)/M^2} \sin^2\theta_1 \right] = \\ & = R(M^2) - 4m_s \langle 0 | \bar{s}s | 0 \rangle + \frac{3m_s^2}{2\pi^2} M^2 E_0 \left(\frac{s_0}{M^2} \right) L^{-8/9} - 6\alpha_s \frac{m_s m_0^2}{\pi M^2} \langle 0 | \bar{s}s | 0 \rangle. \end{aligned} \quad (41)$$

Take now the difference of (39) and one half of (40). The corresponding sum rule is:

$$\begin{aligned}
& -\frac{3}{\sqrt{2}}m_{\eta'}^2 e^{-m_{\eta'}^2/M^2} f_1 f_8 \left[\sin\theta_8 \cos\theta_1 - \frac{m_{\eta}^2}{m_{\eta'}^2} e^{(m_{\eta'}^2 - m_{\eta}^2)/M^2} \cos\theta_8 \sin\theta_1 \right] = \\
& = -4m_s \langle 0 | \bar{s}s | 0 \rangle + \frac{3m_s^2}{2\pi^2} M^2 E_0 \left(\frac{s_0}{M^2} \right) L^{-8/9} - 3\alpha_s \frac{m_s m_0^2}{\pi M^2} \langle 0 | \bar{s}s | 0 \rangle. \quad (42)
\end{aligned}$$

The theoretical value of θ_1 found in [9, 10, 11] is small: $\theta_1 = -(2.7^0 - 4^0)$. (The phenomenological value [10, 11] is slightly higher: $\theta_1 = -9.2^0$). Therefore, with a good accuracy we can put $\theta_1 \approx 0$ in (41),(42). Then (41) determines $f_1^2 \approx f_{\eta'}^2$. M^2 -dependence of f_1^2 is presented on Fig.1. (The numerical values $\langle 0 | \bar{s}s | 0 \rangle = -1.11 \cdot 10^{-2} \text{ GeV}^3$ and $m_s(1 \text{ GeV}) = 150 \text{ MeV}$ were used). From the curve in Fig.1 the estimation follows:

$$f_{\eta'}^2 = (3.2 \pm 0.6) \cdot 10^{-2} \text{ GeV}^2, \quad f_{\eta'} = 178 \pm 17 \text{ MeV}. \quad (43)$$

The ratio of (42) to (41) gives the value of mixing angle θ_8 . In the approximation $\theta_1 \approx 0$ and at $f_8/f_1 = 1.12$ [9, 10, 11] it is equal:

$$\theta_8 = -(17.0 \pm 5.0)^0. \quad (44)$$

The account of $\theta_1 = -2.7^0$ changes θ_8 to

$$\theta_8 = -(18.8 \pm 5.0)^0. \quad (45)$$

The values of $f_{\eta'}$ and θ_8 mixing angle are in an agreement with ones found in [9, 10, 11] from the low energy effective theory or phenomenology. They are correspondingly:

$$\begin{aligned}
& \text{theory:} \quad f_{\eta'} = 151 \text{ MeV}; \quad \theta_8 = -21^0. \\
& \text{phenom.:} \quad f_{\eta'} = 153 \text{ MeV}; \quad \theta_8 = -21^0. \quad (46)
\end{aligned}$$

4 Q^2 -dependence of $\chi(Q^2)$.

Using OPE for $P_L(Q^2)$ – (20) and (13), the Q^2 -dependence of $\chi(Q^2)$ at high Q^2 can be found. Since, by definition of $\chi(Q^2)$, the perturbative part should be subtracted, the first term in the rhs of (20) is omitted and we have:

$$\begin{aligned}
\chi(Q^2) = & -\frac{\alpha_s}{16\pi} \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle \left(1 + \frac{\varepsilon}{Q^2} \right) - \frac{15}{128} \pi \alpha_s \frac{1}{Q^4} \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle^2 - \\
& -\frac{1}{2} n_0 Q^4 \rho_c^4 K_2^2(Q\rho_c), \quad (47)
\end{aligned}$$

where ε parameterizes the 6-dimensional gluonic condensate contribution (see (27)) and (23) was used. $\chi(Q^2)$ (47) is plotted in Fig.2. It is instructive to compare $\chi(Q^2)$ at large Q^2 with $\chi(Q^2)$ at low Q^2 found in [12]:

$$\chi(Q^2) = \chi(0) - \chi'(0)Q^2 - \frac{1}{8}f_\pi^2 Q^2 \left[\left(\frac{m_u - m_d}{m_u + m_d} \right)^2 \frac{m_\pi^2}{Q^2 + m_\pi^2} + \frac{1}{3} \frac{m_\eta^2}{Q^2 + m_\eta^2} \right], \quad (48)$$

where

$$\chi(0) = \frac{m_u m_d}{m_u + m_d} \langle 0 | \bar{u}u | 0 \rangle \quad (49)$$

and the last term represents the contributions of π^0 - and η -mesons. The curve of the low Q^2 behaviour of $\chi(Q^2)$ (48) is also plotted in Fig.2, for $\chi'(0)$ it was chosen the value found in [1]: $\chi'(0) = 2.3 \cdot 10^{-3} \text{ GeV}^2$. As is seen from Fig.2, both curves matches rather well in the domain $Q^2 \approx 0.4 - 1 \text{ GeV}^2$.

5 Discussion. Comparison with results of other works.

As was mentioned in the Introduction, instantons are the most plausible QCD objects for description of physical η' channels, or, what is equivalent, of the longitudinal part of singlet axial vector current correlator. The calculations of the pseudoscalar current $j_5 = \bar{q}\gamma_5 q$ correlator with η' quark content performed by Shuryak and Verbaarschot [17] and Schäfer [18] in the framework of various instanton models demonstrated that this correlator is described in an agreement with its phenomenological coordinate dependence up to distances $x \sim 0.3$ fermi (and in some of them even up to larger ones). Therefore, beforehand, we could expect that the instanton model is suitable for consideration of the problem in view. However, in this paper we used the simplest version of the instanton model – the instanton liquid approximation with instanton density given by the Shuryak model (21). For this reason the accuracy of our results is limited.

Since the main contribution (about 80 %) to the sum rules from which f_0^2 and $f_{\eta'}^2$ were found comes from the instanton term, the main uncertainty is caused by the instanton parameters n_0 and ρ_c . These parameters were taken from the best fit to various hadronic correlators, as well as some other QCD objects, like gluonic condensate performed by Shuryak and his collaborators [6]. Possible uncertainties are included into the errors. The errors in the determination of the mixing angle θ_8 are smaller, because instantons do not contribute to the rhs of (42). If for f_1 and f_8 their phenomenological values will be taken: $f_1 = 1.28f_\pi$, $f_8 = 1.15f_\pi$ [9, 10, 11] instead of $f_1 = f_{\eta'}$ found from the sum rule, then we would have from (42) (at $\theta_1 = -2.7^\circ$):

$$\theta_8 = -(26.5 \pm 3.5)^\circ. \quad (50)$$

The obtained above value of $f_{\eta'}$ (43) is a bit higher than the low energy data (46), but, taking in mind the uncertainties, one may consider the agreement as satisfactory.

The α_s -corrections were not accounted in our calculation. They are essential in the case of the first term in the rhs of (27) [19], but this term contributes only 5% to the total result. The α_s -corrections to gluonic condensate contributions are masked by uncertainties

in higher order gluonic condensates. Among the terms proportional to m_s and m_s^2 , the α_s -corrections appear only to the two last terms in (34), not to the main, proportional to m_s , term $-4m_s\langle 0 | \bar{s}s | 0 \rangle$. They do not much influence the value of the mixing angle. There are also instanton corrections to the term $\sim m_s^2$ of the same order of magnitude. The account of all these corrections to the $\sim m_s^2$ terms is the subject of further study. We believe that after their account the value of the mixing angle will be still in the limits of accepted above errors.

The slopes in M^2 of the left and right hand sides of (41) are different – positive in the lhs and negative in the rhs. For this reason it is impossible to determine $m_{\eta'}$ by differentiation of (41) in M^2 as it was done sometimes in the QCD sum rule approach: whereas the sum rule is satisfactory, its derivative is not.

In their recent paper Narison, Shore and Veneziano [19] studied the problem of determination of $\chi'(0)$ or $f_{\eta'}$ in the framework of the standard QCD sum rule approach with no account of instantons. Their result for $\chi'(0)$ or $f_{\eta'}^2$ is essentially – 3-4 times – smaller than ours. The principal difference from the presented here calculations (besides instanton contribution) is that the authors of [19] have chosen much larger values of the continuum threshold s_0 and of the effective Borel parameters: $s_0 = 6 \text{ GeV}^2$ and $M^2 \sim 3 - 4 \text{ GeV}^2$. Therefore, they accepted the model of hadronic spectrum in $J^{PC} = 0^{-+}$ flavour singlet channel with a gap between η' -meson mass and 2.5 GeV . However, there are at least 3 resonances with η' quantum numbers between η' mass and 2.5 GeV . That is why we think that such a model is not acceptable physically. The other drawback of the sum rule used in [19] (eq.(D.11) of [19], which is similar to (41), but without $\eta - \eta'$ mixing) is that the main contribution to the sum rule comes from the terms proportional to m_s and m_s^2 . These terms comprise 60% of the final answer. This means that SU(3) flavour symmetry is badly violated in the sum rule [19] in contradiction with experiment. Moreover, if we introduce $\eta - \eta'$ mixing (what was not done in [19]), we may calculate the $\eta - \eta'$ mixing angle, representing the phenomenological side of the sum rule by (39),(40). Then the result for the mixing angle θ_8 is: $\theta_8 \approx 45^\circ$, i.e. η' is not mainly flavour singlet and η is not mainly octet – in evident contradiction with experiment.

In conclusion, we have shown that the instanton model even in its simplest version describes reasonably well the properties of the topological density correlator – $\chi'(0)$ and $\chi(Q^2)$ at large Q^2 , the values of the η' coupling constant and the $\eta' - \eta$ mixing angle.

This work was supported in part by RFBR grant 97-02-16131.

References

- [1] Ioffe B.L., Oganesian A.G. // Phys.Rev.D. 1998. vol.57. p.R6590.
- [2] Ioffe B.L. // Lecture at St.Petersburg Winter School on Theoretical Physics, Febr.23-28, 1998, preprint ITEP 7-98, Surveys in High Energy Physics, in press.
- [3] Crewther R.J. // Phys.Lett. 1977. vol.70 B. p.349.
- [4] Ioffe B.L., Khodzhamirian A.Yu. // Yad.Fiz. 1992. vol.55. p.3045.
- [5] Shuryak E.V. // Nucl.Phys.B. 1982. vol.203. p.93. 1983. vol.214. p.237.
- [6] Schäfer T., Shuryak E.V. // Rev.Mod.Phys. 1998. vol.70. p.323.
- [7] Geshkenbein B.V., Ioffe B.L. // Nucl.Phys.B. 1980. vol.166. p.340.
- [8] Novikov V.A., Shifman M.A., Vainshtein, A.I., Zakharov V.I. // Phys.Lett. 1979. vol.86B. p.347.
- [9] Leutwyler H. // Nucl.Phys.Proc.Suppl. 1998. vol.64. p.223.
Kaiser R., Leutwyler H. hep-ph/9806336.
- [10] Feldmann Th., Kroll P., Stech B. // Phys.Rev.D. 1998. vol.58. p.114006.
- [11] Feldmann Th., Kroll P., Stech B. hep-ph/9812269.
- [12] Ioffe B.L. Preprint FZ-IKP(TH)-1998-28, hep-ph/9811217, Phys.At.Nucl., in press.
- [13] Reinders R.J., Rubinstein H.R., Yazaki H. // Phys.Lett. 1984. vol.138B. p.425.
- [14] Novikov V.A., Shifman M.A., Vainshtein A.I., Zakharov V.I. // Nucl.Phys.B. 1981. vol.191. p.301.
- [15] Ioffe B.L. // Nucl.Phys.B. 1981. vol.188. p.317; *ibid*, vol.191. p.591.
- [16] Belyaev V.M., Ioffe,B.L. // JETP. 1982. vol.83. p.876.
- [17] Shuryak E.V., Verbaarschot J.J.M. // Nucl.Phys.B. 1993. vol.410. p.37,55.
- [18] Schäfer T. // Phys.Lett. 1996. B.389. p.445.
- [19] Narison S., Shore G.M., Veneziano G. // Preprint CERN-TH/98-385, hep-ph/9812333.

Figures.

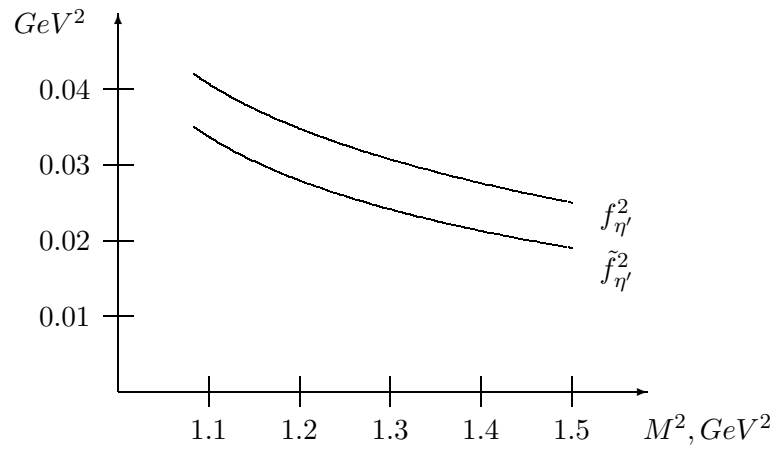


Figure 1.

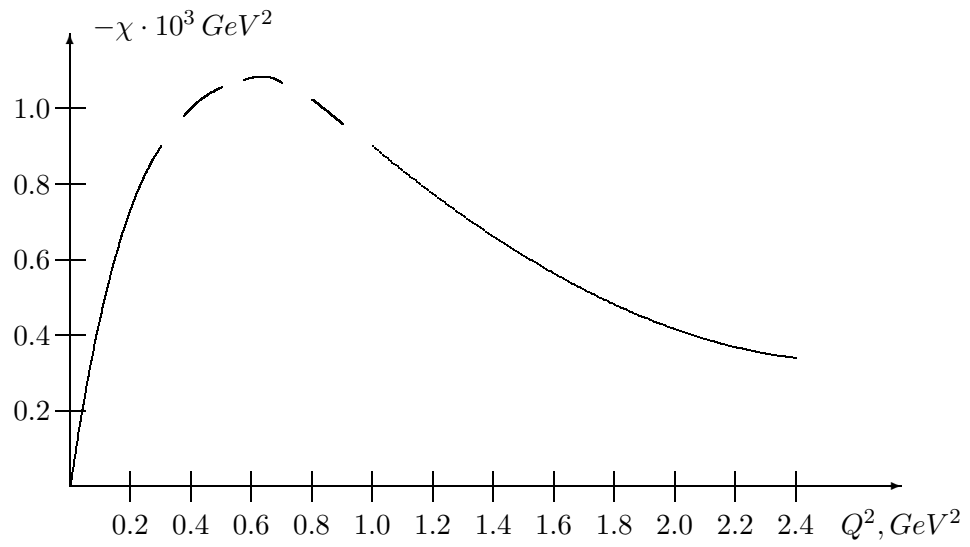


Figure 2.

Figure captions.

Figure 1. Functions $\tilde{f}_{\eta'}^2(M^2)$, $f_{\eta'}^2(M^2)$ determined by equations (27), (41) correspondingly. In (41) $\theta_1 = 0$.

Figure 2. Function $\chi(Q^2)$ at low and large Q^2 – solid lines (see equations (48) and (47) correspondingly). The dashed line represents matching curve drawn by hand.